# **Lesson Objectives**

* 1. The Basics of an Exponential Function
  2. Graph an Exponential Function (use calculator!)
  3. Evaluating a function using the Natural base, *e*
  4. Applications Involving Compound Interest
     1. Regular compounding
     2. Continuous compounding

# The **Basics** of an **Exponential Function**

An **exponential function** is of the form: ,

Where *C* is called the ***initial amount*** (starting amount) and is the *y*-intercept, and

*a* is called the **growth factor** (if ***a* > 1**) or the **decay factor** (if **0 < *a* < 1**)

# The **Graph** of an **Exponential Function**

The graph on an exponential function has two general types, depending on whether it is growth or decay.

|  |  |
| --- | --- |
| **Exponential Growth** (*a* > 1) | **Exponential Decay** (0 < *a* < 1) |
| This is a generic graph of an exponential growth function (a>1) of y=Ca^x. It is an overall increasing function The left tail goes near horizontal above the x-axis towards x=-infinity (horizontal asymptote is y=0). The right tail increases without bound - it just keeps going up and up and up. | This is a generic graph of an exponential decay function (0<a<1) of y=Ca^x. It's an overall decreasing function. As you keep going left toward x=-infinity, the left tail is going up and up and up without bound. The right tail is going horizontal above the x-axis as you go toward x=infinity (horizontal asymptote y=0). |
| **INCREASING** | **DECREASING** |

(go on to the next page)

* **EXAMPLE:** Sketch a graph of *y* = *f*(*x*). [5.3.61]

|  |  |  |  |
| --- | --- | --- | --- |
| **A.** | **B.** | **C.** | **D.** |
| This is a graph of what appears to be the exponential function y=-3(1/8)^x with a viewing window of [-5,5,1] by [-50,50,5] | This is a graph of what appears to be the exponential function y=3(8)^x with a viewing window of [-5,5,1] by [-50,50,5] | This is a graph of what appears to be the exponential function y=3(1/8)^x with a viewing window of [-5,5,1] by [-50,50,5] | This is a graph of what appears to be the exponential function y=-3(8)^x with a viewing window of [-5,5,1] by [-50,50,5] |

. The base (1/8) is between 0 and 1, so this is a **DECAY** function (decreasing).

You can easily verify the correct graph using your graphing calculator – PLEASE do this!!

|  |  |  |
| --- | --- | --- |
| This is a screenshot from the Texas Instruments TI-84 Plus graphing calculator. Description is in the text below this picture.   * + - 1. Press **Y=** button and enter your function. | This is a screenshot from the Texas Instruments TI-84 Plus graphing calculator.  Graphing window settings are Xmin=-5, Xmax=5, Xscl=1, Ymin=-5, Ymax=6, Yscl=1 (don't worry about any of the other information on that screen)   1. Press **WINDOW** button to adjust graph settings. Then press **GRAPH** button. | This is a screenshot from the Texas Instruments TI-84 Plus graphing calculator. It is the graph of Y1=3(1/8)^X on a viewing window of [-5,5,1] by [-50,50,5]   1. This is an exponential decay function. Correct answer is: **C** |

# Evaluating a Function using the **Natural** base, ***e***

The **natural** base, ***e***, is an irrational number (similar to pi, or *π*).

The value of it is *e* ≈ 2.718281828…

To do graphs and/or calculations with the **natural** base ***e***, you can use your calculator.

The button for *e* can be found in two places:

picture of the LN button on the Texas Instruments TI-84 Plus graphing calculator.
When you press the 2ND key with the divide button, you access the "e^x"  above the LN key (used for *ex*) or picture of the "divide" button on the Texas Instruments TI-84 Plus graphing calculator.
When you press the 2ND key with the divide button, you access the "e"  above the divide key

* **EXAMPLE:** Approximate *f*(*x*) to four decimal places. [5.3.49]

*x* = – 1.4 Use your **calculator** for this one!

There are 2 ways to do this:

direct substitution (w/parentheses) or the “go to the STO→” method

**Direct substitution**:

Screenshot from a Texas Instruments TI-83 Plus calculator or TI-84 Plus calculator in "classic" mode.
It is the calculation:
(1/4)*((e^(-1.4)-(e^(-(-1.4))))
which returns the value
-0.9521507507 or Screenshot from a Texas Instruments  TI-84 Plus calculator in "mathprint" mode.
It is the calculation:
(1/4)*((e^(-1.4)-(e^(-(-1.4))))
which returns the value
-0.9521507507 rounds to **– 0.9522**

“**Go to the STO→**” method (plug in – 1.4 for *x* in calculator)

picture of the buttons from Texas instruments TI-83 Plus or 84 Plus calculator.
Sequence is:
-1.4 then STO button then X,T,theta,n button, then ENTER button Screenshot from Texas Instruments TI-84 Plus graphing calculator after pressing the sequence:
-1.4 then STO button, then X button, then ENTER button, returning the value:
-1.4

Screenshot from Texas Instruments TI-84 Plus graphing calculator in "classic" mode.
Screen reads:
(1/4)(e^x-e^(-x) and returns the display:
-0/9521507507 or Screenshot from Texas Instruments TI-84 Plus graphing calculator in "mathprint" mode.
Screen reads:
(1/4)(e^x-e^(-x) and returns the display:
-0/9521507507 same answer

* **EXAMPLE:** A sample of 250 grams of a radioactive substance decays according to the function , where *t* is the time in years. How much of the substance will be left in the sample after 30 years? Round your answer to the nearest whole gram.

[\*Lial 10.6-30, Q10]

Define your variables.

*A*(*t*): **amount of substance** and *t*: **time in years**

To find out how much substance is left after 30 years (*t* = 30), calculate ***A*(30)**, which simply means plug in 0 into the given function formula

(Use calculator).

**Direct Substitution**: Screenshot from Texas Instruments TI-84 Plus graphing calculator in "Math Print" mode.
Screen shows 250e^(-0.045*30) with an answer returned as 62.89463826. or Screenshot from Texas Instruments TI-83 Plus or TI-84 Plus graphing calculator in "Classic" mode.
Screen shows 250e^(-0.045*30) with an answer returned as 62.89463826.

“**Go to the STO→”** Screenshot from Texas Instruments TI-84 Plus graphing calculator in "Math Print" mode after pressing the sequence:
30 then STO button, then ALPHA button followed by 4 button to get the letter T, then ENTER button, returning the value: 30.
Then followed by 250e^(-0.046T) returning the value 62.89463826 or Screenshot from Texas Instruments TI-83 Plus or TI-84 Plus graphing calculator in "Classic" mode after pressing the sequence:
30 then STO button, then ALPHA button followed by 4 button to get the letter T, then ENTER button, returning the value: 30.
Then followed by 250e^(-0.046T) returning the value 62.89463826

Answer: after 30 years, the amount of substance is approximately **63** grams.

# Applications Involving **Compound Interest**

## **Regular** Compoundings **Formula:**

Where: ***A*** is the future value, or final value (**amount**)

***P*** is the **principal** (initial amount, starting amount, deposit, etc.)

***r*** is the interest **rate**, converted from percent to decimal (just divide by 100)

***n*** is the number of interest-compoundings per year:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***n* = 1** | ***n* = 2** | ***n* = 4** | ***n* = 12** | ***n* = 365** |
| annually or yearly | semi-annually | quarterly | monthly | daily |

***t*** is the time in years

* **EXAMPLE:** Use the compound interest formula to determine the final value of the given amount. $1,000 at 15% compounded semiannually for 8 years[5.3-21]

***P* = 1000 *r* = 0.15 *n* = 2 *t* = 8**

**$1000 Principal rate is 15% semiannually 8 years**

Using the Compound Interest formula:

Substitute your given information:

Use your calculator to compute the final amount:

Screenshot from Texas Instruments TI-83 Plus or TI-84 Plus graphing calculator in "Classic" mode after pressing the sequence:
1000(1+0.15/2)^(2*8) returning the value
3180.793154 Screenshot from Texas Instruments TI-84 Plus graphing calculator in "Math Print" mode after pressing the sequence:
1000(1+0.15/2)^(2*8) returning the value
3180.793154

Because it’s money, it rounds to 2 decimal places: **$3180.79 Answer**

## **Continuous Compounding Formula:**

Where: ***A*** is the future value, or final value (**amount**)

***P*** is the **principal** (initial amount, starting amount, deposit, etc.)

***r*** is the interest **rate**, converted from percent to decimal (just divide by 100)

***t*** is the time in years

* **EXAMPLE:**  Use the compound interest formula to determine the final value of the given amount. $400 at 6% compounded continuously for 6 years [5.3.103]

***P* = 400 *r* = 0.06continuous compounding *t* = 6**

Using the Continuous Compound Interest formula:

Substitute your given information:

Use your calculator to compute the final amount:

Screenshot from Texas Instruments TI-83 Plus or TI-84 Plus graphing calculator in "Classic" mode after pressing the sequence:
400e^(0.06*6) returning the value
573.3317658 Screenshot from Texas Instruments TI-84 Plus graphing calculator in "Math Print" mode after pressing the sequence:
400e^(0.06*6) returning the value
573.3317658

Because it’s money, it rounds to 2 decimal places: **$573.33 Answer**

Sources Used:

1. MyLab Math for *Algebra for College Students*, 8th Edition, Lial, Pearson Education Inc.
2. MyLab Math for *College Algebra with Modeling and Visualization*, 6th Edition, Rockswold, Pearson Education Inc.
3. Wabbitemu calculator emulator version 1.9.5.21 by Revolution Software, BootFree ©2006-2014 Ben Moody, Rom8x ©2005-2014 Andree Chea. Website <https://archive.codeplex.com/?p=wabbit>